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AN

# INTRODUCTION
TO THE
# LUNAR METHOD
OF FINDING THE
# LONGITUDE
# IN A SHIP AT SEA:

HAVING
The LATITUDE of the PLACE,
The PREDICTED DISTANCES,
The LOGARITHMIC TABLES,
LINEAR or PROPER TABLES,
AND
COTEMPORARY OBSERVATIONS
OF THE
SUN AND MOON,
OR
MOON AND STARS.

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M,DCC,XC.

# PREFACE.

WHEN an Obſerver in a Ship at Sea obſerves the Sun and Moon in the Day-time, or Moon and Stars in the Night, and thereby calculates the Longitude of the Ship's Place, this diſcovery is ſaid to be by the Lunar Method of finding the Longitude at Sea. In order to practiſe this with ſucceſs, two things are neceſſary, namely: firſt, that Obſervations be accurately made; ſecondly, that they are correctly calculated. To both theſe, the reader is regularly introduced, and to ſeveral new Improvements, by this work.

In this Lunar Method, the principal Difficulties are in the Calculations for Refraction and Parallax; this part I have conſidered anew from it's original principles, in a manner different from that in which it hath been hitherto conſidered by other writers on this ſubject, and have therefrom deduced two general modes of Operation, which are mutual Proofs to each other, or Verifications, by the uſe of the ſame Tables. Theſe are here called Firſt and Second Methods for Refraction and Parallax, and are illuſtrated by Twenty Examples, which exerciſe as great a Variety as can be expected to happen in the practice of this Science. When Obſervations have been made under the beſt circumſtances for avoiding the Error through a variable Refraction, that may be ſuppoſed to happen, a Calculation for Refraction and Parallax, may be made by either of theſe Methods, in three Minutes of time, and a more expeditious Method that is true, cannot be expected, or two that are ſo eaſily and truely Proofs or Verifications to each other.

By keeping to the plan which was originally begun by me, many Years ſince, and which has been many Years practiſed with eaſe and ſucceſs at all parts of the World, the Calculations for Refraction and Parallax are now become ſhortened, and a correct allowance may eaſily be made for Refraction from theſe principles, whatever be its variable ſtate at any part of the Earth or Sea.

The apparent motions and ſituations of the Celeſtial Bodies at different places, are ſuch, that conſiderable advantages might occur by a choice of proper times for the Obſervations; but this is impracticable, on account of the uncertainty of having favourable weather, when the moſt advantageous Altitudes and Diſtances ſhould be taken; and therefore as things now are, it may be conſidered as abſolutely neceſſary, that Obſervations for the Longitude at Sea by the Lunar Method, ſhould be readily made and eaſily calculated univerſally, at all times and places, when the Luminaries are ſufficiently elevated, ſo as to occaſion no Errors through a greater or leſs Refraction than is ſuppoſed near the Horizon. How to allow for ſuch Errors or correct them, has been directed in my former works; but the Second Method in this Work, may be conſidered as of much greater utility and extent.

As this work has a Second Method for Refraction and Parallax, ſo has it a Second Method for the Time at the Ship. The Firſt Method for Ship's Time finds the Co-ſine of Half the Ship's Time from Noon. The ſecond Method for Ship's Time, finds the Sine of Half the Ship's Time from Noon. The ſmall Difference there is between theſe Operations, makes them the eaſier to be underſtood and remembered, and the Correctneſs with which the Hour-angle from Noon is computed by either of theſe Methods nearly to the Meridian and to the Zenith, at places where the Celeſtial Bodies happen to be nearly vertical, cannot but render either of theſe Methods preferable to others which are much leſs perfect.

A Third Method for the effects of Refraction and Parallax in diſtance, has been long ſince publiſhed in my other works, by finding the ſpherical Angles at the Sun and Moon, or Moon and Star. That and the two former Methods, may be conſidered as general ones, eaſily applicable whether the Refraction near the Horizon be greater or leſs than in the temperate Zones, if the Difference be known, and therefore are proper to be uſed in the Lunar Method of finding the Longitude at Sea. Theſe three are ſupported by the Principles of Aſtronomy,

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AN

# AN
# INTRODUCTION
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## SECTION I.

### *Of Latitude, at a Place of Obſervation for the Longitude.*

IN Order to practiſe the Lunar Method of finding the Longitude at Sea with the greateſt ſucceſs, the Obſerver ſhould be provided with ſuch Inſtruments and Tables as are beſt adapted for that purpoſe.

The Sextant to be uſed, ſhould be well adjuſted, and capable of taking Angles to the neareſt Minute of a Degree, and the Logarithmic Tables ſhould be adapted for computing to the like accuracy. The auxiliary Tables under whatever Name (whether logiſtic or proper) to be applied with the Logarithms, ſhould be adapted for computing to the neareſt Second of a Degree, and the Computations made to that accuracy.

The Obſerver ſhould have ſuch Aſſiſtants as are proper and may be depended on, otherwiſe he ſhould take all the Obſervations himſelf. When the Obſervations are made, he ſhould regulate them in the beſt manner, by exterminating thoſe that are very erroneous and retaining their proper ſubſtitutes, as from the probability of events, the Corrections will bring the anſwer nearer to the truth.

If the Latitude be known at the time of the Obſervations for the Longitude, there will be no need of any thing but the three cotemporary Obſervations, but if the Latitude be not known, it will be neceſſary to find it for that time by any Method whatever that will determine it.

At all times, when a Journal has been regularly kept, the Latitude will be ſhewn for the preceding Noon, and the Courſes and Diſtances after, will give the Difference of Latitude. This gives the Latitude at the time of Obſervation for the Longitude. Otherwiſe, if no Journal has been kept, two Altitudes and a ſhort Elapſed-time gives the Latitude. In the Night, the Latitude is determined from an additional Altitude of one which is not the zodiacal Star. Farther,

It may be obſerved, that the Lunar Method of finding the Longitude at Sea, in its moſt perfect ſtate, is no way dependant on any previous knowledge of the Latitude; for it takes both the Latitude and the Longitude, wholly by three Obſervations of the Celeſtial Bodies made at or near the ſame inſtant of time; but as the Latitude by that Method requires very exact Altitudes and other Niceties beſides a Calculation, whilſt it may otherwiſe be more eaſily had, there is leſs need of deducing it from cotemporary Obſervations made for the Longitude, except in caſes of neceſſity; and inſtead thereof it will be beſt for general practice to take it from the Journal Entries, as they are made up to the preceding Noon and carried on to the time of the Obſervations for the Longitude. Thus the Latitude will be known near the Truth, at all times during a long Voyage in a Ship at Sea.

## II. *Of the Predicted Diſtances.*

In the Lunar Method of finding the Longitude at Sea, the true Diſtance of two Celeſtial Bodies, is the Number of Degrees, Minutes, and Seconds, in the arch of a Great Circle of the Heavens, which the Centre of one of thoſe Bodies is from the Centre of the other. The apparent Diſtance of the Centres, or the Limbs, is what it appears to be at an Eye on or near the Surface of the Earth or Sea. The Difference between the true and apparent Diſtance is the Parallax in Diſtance.

In an Ephemeris, the predicted three hourly Diſtances of the Sun and Moon or Moon and Stars, are true Diſtances of their Centres, at the Solar-times mentioned, and for the Ephemeris's Meridian, throughout the Year. Theſe Predictions may be depended on as being calculated according to the moſt accurate Obſervations and lateſt Improvements.

Predicted Diſtances of the Sun and Moon, and of the Moon and zodiacal Stars, are abſolutely neceſſary in the Lunar Method of finding the Longitude at Sea. The Moon's Horizontal Parallax muſt alſo be predicted for the times of Obſervation, to this the Moon's horizontal Semidiameter will be analogous and thereby be eaſily found. The Sun's true Declination for the time of Obſervation muſt likewiſe be predicted, and for the Night Obſervation the Sun's Right Aſcenſion, and the Right Aſcenſions and Declinations of the zodiacal Stars, from which the Moon's Diſtances are to be taken, and the Right Aſcenſions and Declinations of other fixed Stars, or the Primary Planets, that are to be uſed for determining the Time at the Ship.

### III. *Of the Logarithmic Tables.*

The Logarithmic Tables have been ſo fully deſcribed in the Book itſelf, that an account of them here ſeems unneceſſary, except concerning their ſuperior excellence, in Calculations for the Longitude at Sea.

When Obſervations have been made for determining the Longitude by the Lunar Method, it is the Logarithmic Tables only, that can eaſily and correctly give the effect of Refraction to any ſuppoſed ſtate of that at the Horizon; and the effect of Parallax eaſily and truely, according to any ſuppoſed Horizontal Parallax of the Moon.

When the Celeſtial Bodies are near the Meridian, and Altitudes of them have been correctly taken, it is theſe Tables only, that can eaſily and truely give the Reſults, which ariſe from the properties of the original Spherical Triangles. Theſe are the Foundations on which the Diſcovery of the Longitude depends, and if inſtead of theſe Methods, erroneous Methods are ſubſtituted, there is Error.

When the Obſervations for the Longitude have been made, and are to be read on the Inſtruments, they are to the ſmalleſt Subdiviſions that can be diſcerned, even by help of an Eye-Glaſs, and after that, there may be a nearer approach toward the Truth, by taking the Means and correcting them, as is demonſtrable from the Probability of Events. The Logarithmic Tables retain and calculate theſe approaches toward the truth with eaſe and certainty, from four to five or ſix places of Figures, but Delineations with unequal Subdiviſions, / hardly hold good to three places of Figures, which in many caſes is not a hundreth part of the accuracy that is wanted, beſides the Errors that will evitably be in the Delineations. Inſtances of this kind are numerous, in the compoſitions and analyſis of the Sphere.

## IV. *Of Linear and Proper Tables.*

A Linear Table is formed chiefly of Lines, and a Numerical Table is formed chiefly of Numbers. A Proper Table is one deſigned for ſhortening Calculations.

In Linear Tables, when they are made to expreſs Sines, Tangents, or other Numbers increaſing or decreaſing by unequal Differences, it is frequently hard to read three places of figures by them, where four, five or ſix are wanted, in their moſt favourable parts. In their leaſt favourable parts, the Subdiviſions cannot be inſerted, and if they could they could not be diſtinguiſhed.

In the Lunar Method of finding the Longitude, the effect of Parallax, at a Medium, requires four Figures to expreſs its reſult only. The greateſt effect of Refraction in Diſtance, compared with that of Parallax in Diſtance, where the Lunar Method ſhould begin to be practiſed, is but as one to ten nearly; afterward as a Medium but as one to fourty nearly. The effect of Refraction in Diſtance being ſo ſmall, and that of Parallax in Diſtance ſo great, I ſubſtituted Linear Tables, for finding the effect of Refraction in Diſtance, and Numerical Tables for finding the effect of Parallax in Diſtance, thereby preſerving nearly an equal Accuracy in both, which otherwiſe would have been very defective, by applying Lines for the Parallax in Diſtance. This was fully expreſſed in my Preface to the Linear Tables, more than ſeven years ſince.

The Conſtruction, Deſcription and Application of thoſe Linear Tables and Proper Tables, have been ſeveral times printed in my works. A conciſe account of their principal uſe is as follows.

## V. *Linear and Proper Tables Deſcribed.*

Linear Table I. has three kinds of Lines. The leſſer Altitude is to be found from four Degrees to fifty Degrees, in the ſtraight Lines drawn parallel to the Length of the Table. The greater Altitude is to be found from four Degrees to ninety Degrees, in the ſtraight Lines drawn perpendicular to the length of the Table. The Number to be taken out, is in the Lines drawn a little curved through the Table, and their Numbers are the ſame from the left to the right end of the Table. There is no need of being more nice than the Eye and Judgment can eaſily determine, concerning Parts of a Degree, or Parts between the Curved Lines, in taking out Numbers from this Table.

Linear Table II. has but one Line; on one Side of which are the Degrees of Diſtance, and on the other Side the Correction, ſubductive or additive.

By the Altitudes, a Number is found in Tables I. and by the Diſtance a Number in Table II. Then it will be thus. To find the Effect of Refraction in Diſtance.

The Co-ar of the Diſtance, added to the Number from Table I. their Sum is a Common Logarithm, whoſe Natural Number, augmented or diminiſhed by the Seconds from Table II. gives the Correction additive for Refraction.

When the two Altitudes will not go into Table I. neither it nor Table II. are to be uſed; but inſtead thereof,

with

with the Diſtance, the additive Correction for Refraction, is to be taken from Table III. Conſequently, in favourable weather, the Obſervations may be frequently made when the Sun and Moon, or Moon and Star's Altitudes, approach near to an equality, and then the Correction for Refraction may be taken from Table III. by Inſpection, beſides which, theſe will be the beſt times for being undeceived, through a variable ſtate of the horizontal Refraction.

Linear Table IV. is a Table for a very ſmall Correction of Parallax. The Parallax in Altitude is in the ſhorteſt ſide, and the Diſtance in the longeſt Side, and in the other Side from the curved Lines are the Seconds. Alſo, with the whole Parallax in Diſtance in the ſhorteſt Side, and the Diſtance in the longeſt Side, from the curved Lines are Seconds; the Difference of theſe Seconds is additive, when the Diſtance is under Ninety Degrees, otherwiſe ſubductive.

Linear Table XIV. is a Scale ſhewing the Parallax in Altitude. The ſlope Lines leading from the Altitudes to the parallel Lines from the Horizontal Parallaxes, meet among the perpendicular Lines, the Parallaxes in Altitude required. This is wholly for the Moon.

Proper Tables VIII. IX. X. XI. are together one Numerical Table, deſigned for finding the effects of Parallax in Diſtance to the neareſt Second of a Degree, under any obſerved Altitudes of Sun and Moon, or Moon and Star, Diſtance of Centres, and Horizontal Parallax of the Moon, being applyed with the Logarithmic Tables. The other Linear and Proper Tables, have their Uſes plainly expreſſed by their Titles and the Explanations, in their Deſcription and Uſe at large.

## VI. *Of Cotemporary Obſervations, of the Sun and Moon, or Moon and Zodiacal Stars.*

When Obſervations of two or more Celeſtial Bodies, are made at or near the ſame inſtant of time, they are ſaid to be cotemporary; therefore, when the Diſtance of Sun and Moon's neareſt Limbs, the Altitude of the Sun's lower Limb, and Altitude of the Moon's lower and upper

Limb are taken at the ſame inſtant, they are cotemporary Obſervations. By one Obſerver they may be partly inferred.

In the Lunar Method of finding the Longitude, the forementioned three cotemporary Obſervations in the Day-time, and the Latitude of the place, are all that is wanted for determining the Longitude. In the Night-time, the Diſtance of a zodiacal Star from the Moon's neareſt or fartheſt Limb, the Altitude of the Star, and the Altitude of the Moon's lower or upper Limb, with the Latitude of the place, are all that is wanted for determining the Longitude. After the Obſervations have been made, the anſwer comes from the application of the Tables.

When theſe Obſervations have been made, they muſt be prepared for a Calculation; the Semidiameter of the Sun is additive, but his Dip and Refraction are ſubductive, this gives the Sun's true Altitude; the like for the Moon when her lower Limb is taken, but when her upper Limb is taken, the Semidiameter, Dip and Refraction are all three ſubductive, this gives the true Altitude of the Moon's Centre, as affected by Refraction and Parallax.

The Diſtance of Sun and Moon's Limbs read on the Sextant, muſt be increaſed by the Moon's Semidiameter at the time of Obſervation, and Seconds for Moon's Altitude according to her Horizontal Parallax, and this gives the obſerved Diſtance of Sun and Moon's Centres. In like manner, when the neareſt Limbs of Moon and Star have been joined by the Sextant, the Moon's Semidiameter and Seconds for her Altitude are both additive; but when the fartheſt Limb of the Moon has been uſed, both her Semidiameter and Seconds for her Altitude are ſubductive, and this gives the obſerved Diſtance of Star and Moon's Centre, here the Diſtance is affected.

Having made the three cotemporary Obſervations, the true Diſtance of Centres whether of Sun and Moon, or Moon and Star, may be found as in the following Examples.

### VII. *Examples and Reſults by Firſt Method for Refraction and Parallax; ſelected from the firſt Publication of the Linear Tables.*

| EXAMPLE I. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 51.° | 28.′ | 35″ | Refract. | 0.° | 1.′ | 28″ | C | 0.° | 20.′ | 19″ |
| Star's Altitude | 24. | 48. | 0 | D | 51. | 30. | 3 | E | 51. | 9. | 34 |
| Moon's Alt. | 12. | 30. | 0 | 1ſt Arc | 0. | 30. | 7 | F | 0. | 0. | 20 |
| Hor. Par. | | 56. | 15 | 2d Arc | 0. | 9. | 38 | P | 51. | 9. | 54 |
| | | | | By a Mean of Seven beſt Methods | | | | | 51. | 9. | 52 |

| EXEMPLE II. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 90.° | 21.′ | 13″ | Refract. | 0.° | 9.′ | 33″ | C | 1.° | 1.′ | 29″ |
| Star's Altitude | 84. | 7. | 6 | D | 90. | 30. | 46 | E | 89. | 29. | 15 |
| Moon's Alt. | 5. | 7. | 8 | 1ſt Arc | 1. | 1. | 29 | F | 0. | 0. | 1 |
| Hor. Par. | 1. | 1. | 48 | 2d Arc | 0. | 0. | 2 | P | 89. | 29. | 16 |
| | | | | By a Mean of Six beſt Methods | | | | | 89. | 29. | 16 |

| EXAMPLE III. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 110.° | 22.′ | 5″ | Refract. | 0.° | 3.′ | 16″ | C | 0.° | 50.′ | 50″ |
| Sun's Altitude | 45. | 33. | 6 | D | 110. | 25. | 21 | E | 109. | 34. | 31 |
| Moon's Alt. | 19. | 43. | 6 | 1ſt Arc | 0. | 57. | 19 | F | 0. | 0. | 1 |
| Hor. Par. | | 57. | 19 | 2d Arc | 0. | 7. | 11 | P | 109. | 34. | 30 |
| | | | | By a Mean of Four beſt Methods | | | | | 109. | 34. | 28 |

*Examples and Reſults by Firſt Method for Refraction and Parallax; ſelected from the firſt Publication of the Linear Tables.*

| EXAMPLE IV. | *Queſtion.* | | | | | *Reſults.* | | | *Reſults.* | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 50° | 8.′ | 41″ | Refract. | 0° | 2.′ | 3″ | C | 0° | 15.′ | 41″ |
| Star's Altitude | 19. | 18. | 5 | D | 50. | 10. | 44 | E | 50. | 26. | 25 |
| Moon's Alt. | 55. | 56. | 5 | 1ſt Arc | 0. | 25. | 48 | F | 0. | 0. | 5 |
| Hor. Par. | 1. | 0. | 5 | 2d Arc | 0. | 41. | 29 | P | 50. | 26. | 30 |
| | | | | By a Mean of Five beſt Methods | | | | | 50. | 26. | 29 |

| EXAMPLE V. | *Queſtion.* | | | | | *Reſults.* | | | *Reſults.* | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 28° | 14.′ | 39″ | Refract. | 0° | 0.′ | 30″ | C | 0° | 6.′ | 56″ |
| Star's Altitude | 20. | 11. | 4 | D | 28. | 15. | 9 | E | 28. | 8. | 13 |
| Moon's Alt. | 18. | 56. | 4 | 1ſt Arc | 0. | 40. | 22 | F | 0. | 0. | 44 |
| Hor. Par. | | 55. | 30 | 2d Arc | 0. | 33. | 26 | P | 28. | 8. | 57 |
| | | | | By a Mean of Four beſt Methods | | | | | 28. | 8. | 57 |

| EXAMPLE VI. | *Queſtion.* | | | | | *Reſults.* | | | *Reſults.* | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 59° | 25.′ | 34″ | Refract. | 0° | 1.′ | 34″ | C | 0° | 43.′ | 44″ |
| Sun's Altitude | 59. | 12. | 5 | D | 59. | 27. | 8 | E | 58. | 43. | 24 |
| Moon's Alt. | 27. | 2. | 5 | 1ſt Arc | 0. | 59. | 48 | F | 0. | 0. | 5 |
| Hor. Par. | | 59. | 58 | 2d Arc | 0. | 16. | 4 | P | 58. | 43. | 29 |
| | | | | By a Mean of Four beſt Methods | | | | | 58. | 43. | 28 |

*Examples and Reſults by Firſt Method for Refraction and Parallax; ſelected from the firſt Publication of the Linear Tables.*

| **EXAMPLE VII.** | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 43.° | 35.′ | 42″ | Refract. | 0.° | 0.′ | 45″ | C | 0.° | 5.′ | 52″ |
| Star's Altitude | 11. | 18. | 6 | D | 43. | 36. | 27 | E | 43. | 30. | 35 |
| Moon's Alt. | 9. | 37. | 6 | 1ſt Arc | 0. | 15. | 24 | F | 0. | 0. | 26 |
| Hor. Par. | | 54. | 42 | 2d Arc | 0. | 9. | 32 | P | 43. | 31. | 1 |
| | | | | By a Mean of Four beſt Methods | | | | | 43. | 31. | 3 |

| **EXAMPLE VIII.** | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 29.° | 24.′ | 46″ | Refract. | 0.° | 0.′ | 31″ | C | 0.° | 27.′ | 16″ |
| Star's Altitude | 64. | 19. | 4 | D | 29. | 25. | 17 | E | 28. | 58. | 1 |
| Moon's Alt. | 49. | 57. | 4 | 1ſt Arc | 1. | 44. | 50 | F | 0. | 0. | 7 |
| Hor. Par. | | 57. | 8 | 2d Arc | 1. | 17. | 34 | P | 28. | 58. | 8 |
| | | | | By a Mean of Four beſt Methods | | | | | 28. | 58. | 9 |

| **EXAMPLE IX.** | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 62.° | 23.′ | 59″ | Refract. | 0.° | 1.′ | 56″ | C | 0.° | 1.′ | 27″ |
| Star's Altitude | 18. | 42. | 6 | D | 62. | 25. | 55 | E | 62. | 27. | 22 |
| Moon's Alt. | 47. | 42. | 6 | 1ſt Arc | 0. | 20. | 36 | F | 0. | 0. | 5 |
| Hor. Par. | | 57. | 6 | 2d Arc | 0. | 22. | 3 | P | 62. | 27. | 27 |
| | | | | By a Mean of Four beſt Methods | | | | | 62. | 27. | 26 |

*Examples and Reſults by Firſt Method for Refraction and Parallax; ſelected from the firſt Publication of the Linear Tables.*

| **EXAMPLE X.** | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 85° | 0.′ | 0″ | Refract. | 0° | 4′ | 54″ | C | 0° | 2.′ | 30″ |
| Star's Altitude | 5. | 0. | 5 | D | 85. | 4. | 54 | E | 85. | 2. | 24 |
| Moon's Alt. | 30. | 0. | 5 | 1ſt Arc | 0. | 5. | 4 | F | 0. | 0. | 2 |
| Hor. Par. | 1. | 0. | 0 | 2d Arc | 0. | 2. | 34 | P | 85. | 2. | 26 |
| | | | | By a Mean of Four beſt Methods | | | | | 85. | 2. | 25 |

| **EXAMPLE XI.** | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 38° | 22.′ | 17″ | Refract. | 0° | 1.′ | 1″ | C | 0° | 5.′ | 5″ |
| Star's Altitude | 12. | 27. | 5 | D | 38. | 23. | 18 | E | 38. | 28. | 23 |
| Moon's Alt. | 20. | 9. | 4 | 1ſt Arc | 0. | 19. | 49 | F | 0. | 0. | 31 |
| Hor. Par. | | 57. | 24 | 2d Arc | 0. | 24. | 54 | P | 38. | 28. | 54 |
| | | | | By a Mean of Four beſt Methods | | | | | 38. | 28. | 55 |

| **EXAMPLE XII.** | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 45° | 40.′ | 14″ | Refract. | 0° | 0.′ | 57 | C | 0° | 1.′ | 28″ |
| Star's Altitude | 15. | 54. | 4 | D | 45. | 41. | 11 | E | 45. | 39. | 43 |
| Moon's Alt. | 21. | 26. | 4 | 1ſt Arc | 0. | 21. | 35 | F | 0. | 0. | 23 |
| Hor. Par. | | 56. | 30 | 2d Arc. | 0. | 20. | 27 | P | 45. | 40. | 6 |
| | | | | By a Mean of Four beſt Methods | | | | | 45. | 40. | 5 |

EXAMPLE XIII.

*Examples and Reſults by Firſt Method for Refraction and Parallax; ſelected from the firſt Publication of the Linear Tables.*

| EXAMPLE XIII. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 65.° | 27.′ | 30″ | Refract. | 0.° | 3.′ | 16″ | C | 0.° | 57.′ | 50″ |
| Star's Altitude | 78. | 18. | 6 | D | 65. | 30. | 46 | E | 64. | 32. | 56 |
| Moon's Alt. | 15. | 21. | 6 | 1ſt Arc | 1. | 5. | 1 | F | 0. | 0. | 1 |
| Hor. Par. | 1. | 0. | 25 | 2d Arc | 0. | 7. | 11 | P | 64. | 32. | 57 |
| | | | | By a Mean of Four beſt Methods | | | | | 64. | 32. | 59 |

| EXAMPLE XIV. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 49.° | 17.′ | 21″ | Refract. | 0.° | 1.′ | 7″ | C | 0.° | 0.′ | 46″ |
| Star's Altitude | 27. | 15. | 5 | D | 49. | 18. | 28 | E | 49. | 17. | 42 |
| Moon's Alt. | 43. | 20. | 5 | 1ſt Arc | 0. | 35. | 30 | F | 0. | 0. | 13 |
| Hor. Par. | | 58. | 50 | 2d Arc | 0. | 34. | 44 | P | 49. | 17. | 55 |
| | | | | By a Mean of Four beſt Methods | | | | | 49. | 17. | 54 |

| EXAMPLE XV. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 35.° | 29.′ | 45″ | Refract. | 0.° | 0.′ | 37″ | C | 0.° | 1.′ | 40″ |
| Star's Altitude | 17. | 40. | 5 | D | 35. | 30. | 22 | E | 35. | 28. | 42 |
| Moon's Alt. | 20. | 34. | 5 | 1ſt Arc | 0. | 29. | 24 | F | 0. | 0. | 33 |
| Hor. Par. | | 56. | 24 | 2d Arc | 0. | 27. | 44 | P | 35. | 29. | 15 |
| | | | | By a Mean of Four beſt Methods | | | | | 35. | 29. | 16 |

EXAMPLE XVI.

*Examples and Reſults by Firſt Method for Refraction and Parallax; ſelected from the firſt Publication of the Linear Tables.*

| EXAMPLE XVI. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 102° | 30′ | 0″ | Refract. | 0° | 2′ | 37″ | C | 0° | 21′ | 20′ |
| Star's Altitude | 15. | 25. | 5 | D | 102. | 32. | 37 | E | 102. | 11. | 17 |
| Moon's Alt. | 27. | 30. | 5 | 1ſt Arc | 0. | 15. | 29 | F | 0. | 0. | 5 |
| Hor. Par. | | 57. | 3 | 2d Arc | 0. | 5. | 51 | P | 102. | 11. | 12 |
| | | | | By a Mean of Five beſt Methods | | | | | 102. | 11. | 10 |

| EXAMPLE XVII. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 33° | 15′ | 0″ | Refract. | 0° | 0′ | 35″ | C | 0° | 0′ | 48″ |
| Star's Altitude | 48° | 20′ | 5 | D | 33. | 15. | 35 | E | 33. | 16. | 23 |
| Moon's Alt. | 64. | 30. | 5 | 1ſt Arc | 1. | 15. | 33 | F | 0. | 0. | 6 |
| Hor. Par. | | 55. | 29 | 2d Arc | 1. | 16. | 21 | P | 33. | 16. | 29 |
| | | | | By a Mean of Four beſt Methods | | | | | 33. | 16. | 30 |

| EXAMPLE XVIII. | *Queſtion.* | | | | *Reſults.* | | | | *Reſults.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diſtance | 56° | 17′ | 44″ | Refract. | 0° | 1′ | 1″ | C | 0° | 22′ | 1″ |
| Star's Altitude | 53. | 13. | 8 | D | 56. | 18. | 45 | E | 55. | 56. | 44 |
| Moon's Alt. | 64. | 38. | 4 | 1ſt Arc | 0. | 58. | 50 | F | 0. | 0. | 0 |
| Hor. Par. | 1. | 1. | 9 | 2d Arc | 0. | 36. | 49 | P | 55. | 56. | 44 |
| | | | | By a Mean of Six beſt Methods | | | | | 55. | 56. | 45 |

*Examples and Reſults by Firſt Method for Refraction and Parallax; ſelected from the firſt Publication of the Linear Tables.*

| EXAMPLE XIX. | *Queſtion.* |  | *Reſults.* |  | *Reſults.* |
|---|---|---|---|---|---|
| Diſtance | 113°. 19′. 26″ | Refract. | 0°. 6′. 16″ | C | 0°. 17′. 19″ |
| Sun's Altitude | 5. 1. 5 | D | 113. 25. 42 | E | 113. 8. 23 |
| Moon's Alt. | 29. 38. 5 | 1ſt Arc | 0. 5. 13 | F | 0. 0. 9 |
| Hor. Par. | 56. 29 | 2d Arc | 0. 12. 6 | P | 113. 8. 14 |
|  |  | By a Mean of Three beſt Methods |  |  | 113. 8. 15 |

| EXAMPLE XX. | *Queſtion.* |  | *Reſults.* |  | *Reſults.* |
|---|---|---|---|---|---|
| Diſtance | 101°. 46′. 43″ | Refract. | 0°. 3′. 19″ | C | 0°. 51′. 46″ |
| Sun's Altitude | 56. 16. 0 | D | 101. 46. 43 | E | 100. 58. 16 |
| Moon's Alt. | 17. 47. 0 | 1ſt Arc | 0. 48. 9 | F | 0. 0. 1 |
| Hor. Par. | 56. 40 | 2d Arc | 0. 3. 37 | P | 100. 58. 15 |
|  |  | By a Mean of Three beſt Methods |  |  | 100. 58. 15 |

Theſe Reſults are as they ſtand in the abovementioned work, and the Means as deduced from the Anſwers by Others, in the ſame Pages. As theſe Solutions were had without Nicety, and come ſo near the Truth, it is evident this Firſt Method is not inferior to any that has been publiſhed.

## VIII. *Examples and Operations by Second Method for Refraction and Parallax.*

# A New Formula for LONGITUDE, having the Linear Tables, &c. By S. DUNN.

N.B. Co. is Remᵣ. to 90°. Supp. is Rem. to 180°. Co-ar. is Co-secant less Index 10. Sine, Tangent, Secant or Co-ar. of Degrees more than 90° is that of Supp. Subtract if not to add. Solar Time is used for the Longitude in Hours & Degrees. Mean Solar Time is that which Clocks & Watches are made to keep nearly at Sea & on Land.

° ′ ″

Distance Limbs =
● Semidiam$^{r}$. =
☽ Semidiam$^{r}$. =
For ☽ Altitude =

° ′ ″

Distance Centres L
For ● & ☽ Alt

add to Dist. Centres

° ′ ″

D. =
● Altitude =
☽ Hor. Par =

° ′ ″

First Arc =

° ′ ″

D. =
☽ Altitude =
☽ Hor. Par =

° ′ ″

Second Arc =

° ′ ″

Corr$^{n}$ for Par$^{x}$. C. =
D. =

Add C. to D. only when the first Arc is the least of the two & at same time D. is less than 90°.

E. =
F. =

° ′ ″

P. =

° ′ ″

Rough Hour at Greenw.
● Altitude =
☽ Altitude =
☽ Hor. Par. =

Co-ar. =
in Table I = 2.

Common Log$^{m}$ = A =
in Table II = B =

= Correction for Refrac$^{n}$ =

Sine =
Co-Secant =
Proper Log$^{m}$ =

Proper Log$^{m}$ =

Tangent =
Co-secant =
Proper Log$^{m}$ =

Proper Log$^{m}$ =

Add the Arcs together if D. exceeds 90°

add 90° if E. is under 90°

True Distance

° ′ ″

1$^{st}$ Hours =
2$^{d}$ Hours =

Diff. f =

H ′ ″

Time at Greenw. past Noon.

° ′

Co-Latitude = Co-ar.
Polar-dist = Co-ar.
● Co-alt =
2)
Half-sum = Sine
Remaind$^{r}$ = Sine
2)
= Co-sine

sub.

Solar Time as per Watch

H ′ ″

= Time at Ship

H ′ ″

= Longitude from Greenwich.

° ′ ″

Ditto =
P. =

Diff$^{ce}$ ſ =

Prop$^{r}$. Log. ſ =
Prop$^{r}$. Log. f =

sub.

Tab. Prop. Log =
Hours add in Degrees or Hours.

| Dip. | Feet. |
|---|---|
| 1′ | 1 |
| 2 | 4 |
| 3 | 8 |
| 4 | 14 |
| 5 | 22 |
| 6 | 32 |
| 7 | 43 |
| 8 | 55 |

| Alt. ° | | Ref$^{n}$. ′ | Alt. ° | | Ref$^{n}$. ′ |
|---|---|---|---|---|---|
| 42 | = | 1 | 5 | = | 9 |
| 24 | = | 2 | 5 | = | 10 |
| 17 | = | 3 | 4 | = | 11 |
| 13 | = | 4 | 4 | = | 12 |
| 11 | = | 5 | 3 | = | 13 |
| 9 | = | 6 | 3 | = | 14 |
| 7 | = | 7 | 3 | = | 15 |
| 6 | = | 8 | 3 | = | 16 |

For Moon's Alt.

| ☽ Alt. ° | | Increase of Semi. ″ |
|---|---|---|
| 10 | = | 3 |
| 20 | = | 5 |
| 30 | = | 8 |
| 40 | = | 10 |
| 50 | = | 12 |
| 60 | = | 14 |
| 80 | = | 16 |
| 90 | = | 17 |

| Hours. | | Degrees. ° |
|---|---|---|
| 3 | = | 45 |
| 6 | = | 90 |
| 9 | = | 135 |
| 12 | = | 180 |
| 15 | = | 225 |
| 18 | = | 270 |
| 21 | = | 315 |
| 24 | = | 360 |

| Month. | Day. | ☉ on the Merid$^{n}$. |
|---|---|---|
| Jan. | 1 | 12.4 |
| | 8 | 7 |
| | 15 | 10 |
| | 22 | 12 |
| Feb. | 1 | 14 |
| | 8 | 15 |
| | 15 | 15 |
| | 22 | 14 |
| Mar. | 1 | 12½ |
| | 8 | 11 |
| | 15 | 9 |
| | 22 | 7 |
| Apr. | 1 | 4 |
| | 8 | 2 |
| | 15 | 11.59 |
| | 22 | 58 |
| May | 1 | 57 |
| | 8 | 56 |
| | 15 | 56 |
| | 22 | 56 |
| Jun. | 1 | 57 |
| | 8 | 59 |
| | 15 | 12.0 |
| | 22 | 2 |
| Jul. | 1 | 3½ |
| | 8 | 5 |
| | 15 | 5½ |
| | 22 | 6 |
| Aug. | 1 | 6 |
| | 8 | 5 |
| | 15 | 4 |
| | 22 | 2½ |
| Sep. | 1 | 11.59½ |
| | 8 | 57 |
| | 15 | 55 |
| | 22 | 52½ |
| Oct. | 1 | 49½ |
| | 8 | 47½ |
| | 15 | 46 |
| | 22 | 44½ |
| Nov. | 1 | 44 |
| | 8 | 44 |
| | 15 | 45 |
| | 22 | 47 |
| Dec. | 1 | 50 |
| | 8 | 53 |
| | 15 | 56 |
| | 22 | 59½ |

Each to the nearest Half Minute.

Published according to Act of Parliament October . 4 . 1783 by Samuel Dunn, Bow's head Court Fleetstreet LONDON.

### EXAMPLE I.

| | | | | |
|---|---|---|---|---|
| Diſtance | 51°. | 28.′ | 35″ | corrected. |
| Star's Altitude | 24. | 48. | 0 | cleared. |
| Moon's Alt. | 12. | 30. | 0 | cleared. |
| Hor. Par. | | 56. | 15 | ſame time. |

### OPERATIONS.

| | | | | |
|---|---|---|---|---|
| 9.8934 | 10.0991 | 0°. | 30.′ | 10″ |
| 10.3773 | 10.6647 | 0. | 9. | 31 |
| 0.5051 | 0.5051 | 0. | 20. | 29 |
| 0.7758 | 1.2689 | 51. | 8. | 6 |
| | | + | 1. | 48 |
| | True Diſtance | 51. | 9. | 54 |

### EXAMPLE II.

| | | | | |
|---|---|---|---|---|
| Diſtance | 90°. | 21.′ | 13″ | corrected. |
| Star's Altitude | 84. | 7. | 6 | cleared. |
| Moon's Alt. | 5. | 17. | 8 | cleared. |
| Hor. Par. | 1. | 1. | 48 | ſame time. |

### OPERATIONS.

| | | | | |
|---|---|---|---|---|
| 9.9999 | 12.2140 | 1°. | 1.′ | 30″ |
| 0.0023 | 11.0358 | 0. | 0. | 0 |
| 0.4643 | 0.4643 | 1. | 1. | 30 |
| 0.4665 | 3.7141 | 89. | 19. | 40 |
| | | + | 9. | 33 |
| | True Diſtance | 89. | 29. | 16 |

### EXAMPLE III.

| | | | | |
|---|---|---|---|---|
| Diſtance | 110°. | 22.′ | 5″ | corrected. |
| Sun's Altitude | 45. | 33. | 6 | cleared. |
| Moon's Alt. | 19. | 43. | 6 | cleared. |
| Hor. Par. | | 57. | 19 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.9719 | 10.4303 | 0°. | 43.′ | 39″ |
| 10.1464 | 10.4719 | 0. | 7. | 11 |
| 0.4970 | 0.4970 | 0. | 50. | 50 |
| 0.6153 | 0.6153 | 109. | 31. | 15 |
| | | + | 3. | 15 |
| | True Diſtance | 109. | 34. | 30 |

### EXAMPLE IV.

| | | | | |
|---|---|---|---|---|
| Diſtance | 50°. | 8.′ | 41″ | corrected. |
| Star's Altitude | 19. | 18. | 5 | cleared. |
| Moon's Alt. | 55. | 56. | 5 | cleared. |
| Hor. Par. | 1. | 0. | 5 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.8852 | 10.0785 | 0°. | 25.′ | 52″ |
| 10.4808 | 10.0818 | 0. | 41. | 33 |
| 0.4765 | 0.4765 | 0. | 15. | 41 |
| 0.8425 | 0.6368 | 50. | 24. | 22 |
| | | + | 2. | 8 |
| | True Diſtance | 50. | 26. | 30 |

### EXAMPLE V.

| | | | | |
|---|---|---|---|---|
| Diſtance | 28° | 14.′ | 39″ | corrected. |
| Star's Altitude | 20. | 11. | 4 | cleared. |
| Moon's Alt. | 18. | 56. | 4 | cleared. |
| Hor. Par. | | 55. | 30 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.6751 | 9.7302 | 0° | 40.′ | 28″ |
| 10.4621 | 10.4888 | 0. | 33. | 31 |
| 0.5110 | 0.5110 | 0. | 6. | 57 |
| 0.6482 | 0.7300 | 28. | 7. | 42 |
| | | + | 1. | 14 |
| True Diſtance | | 28. | 8. | 56 |

### EXAMPLE VI.

| | | | | |
|---|---|---|---|---|
| Diſtance | 59° | 25.′ | 34″ | corrected. |
| Sun's Altitnde | 59. | 12. | 5 | cleared. |
| Moon's Alt. | 27. | 2. | 5 | cleared. |
| Hor. Par. | | 59. | 58 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.9350 | 10.2287 | 0° | 59.′ | 51″ |
| 10.0660 | 10.3425 | 0. | 16. | 6 |
| 0.4773 | 0.4773 | 0. | 43. | 45 |
| 0.4783 | 1.0485 | 58. | 41. | 49 |
| | | + | 1. | 39 |
| True Diſtance | | 58. | 43. | 28 |

### EXAMPLE VII.

| | | | | |
|---|---|---|---|---|
| Diſtance | 43° | 35.′ | 42″ | corrected. |
| Star's Altitude | 11. | 18. | 6 | cleared. |
| Moon's Alt. | 9. | 37. | 6 | cleared. |
| Hor. Par. | | 54. | 42 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.8386 | 9.9788 | 0° | 15.′ | 32″ |
| 10.7079 | 10.7771 | 0. | 9. | 36 |
| 0.5173 | 0.5173 | 0. | 5. | 56 |
| 1.0638 | 1.2732 | 43. | 29. | 46 |
| | | + | 1. | 11 |
| True Diſtance | | 43. | 30. | 57 |

### EXAMPLE VIII.

| | | | | |
|---|---|---|---|---|
| Diſtance | 29° | 24.′ | 46″ | corrected. |
| Star's Altitude | 64. | 19. | 4 | cleared. |
| Moon's Alt. | 49. | 57. | 4 | cleared. |
| Hor. Par. | | 57. | 8 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.6912 | 9.7512 | 1° | 44.′ | 49″ |
| 10.0452 | 10.1161 | 1. | 17. | 33 |
| 0.4984 | 0.4984 | 0. | 27. | 16 |
| 0.2348 | 0.3657 | 28. | 57. | 30 |
| | | + | 0. | 38 |
| True Diſtance | | 28. | 58. | 8 |

### EXAMPLE IX.

| | | | | |
|---|---|---|---|---|
| Diſtance | 62°. | 23.′ | 59″ | corrected. |
| Star's Altitude | 18. | 42. | 6 | cleared. |
| Moon's Alt. | 46. | 42. | 6 | cleared. |
| Hor. Par. | | 57. | 6 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.9475 | 10.2817 | 0°. | 20.′ | 40″ |
| 10.4940 | 10.1310 | 0. | 22. | 4 |
| 0.4986 | 0.4986 | 0. | 1. | 24 |
| 0.9401 | 0.9113 | 62. | 25. | 23 |
| | | + | 2. | 1 |
| | True Diſtance | 62. | 27. | 24 |

### EXAMPLE X.

| | | | | |
|---|---|---|---|---|
| Diſtance | 85°. | 0.′ | 0″ | corrected. |
| Star's Altitude | 4. | 50. | 0 | cleared. |
| Moon's Alt. | 29. | 58. | 0 | cleared. |
| Hor. Par. | 1. | 0. | 0 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.9983 | 11.0580 | 0°. | 5.′ | 4″ |
| 11.0744 | 10.3015 | 0. | 2. | 37 |
| 0.4771 | 0.4771 | 0. | 2. | 27 |
| 1.5498 | 1.8366 | 84. | 57. | 33 |
| | | + | 4. | 56 |
| | True Diſtance | 85. | 2. | 29 |

### EXAMPLE XI.

| | | | | |
|---|---|---|---|---|
| Diſtance | 38°. | 22.′ | 17″ | corrected. |
| Star's Altitude | 12. | 27. | 5 | cleared. |
| Moon's Alt. | 20. | 9. | 4 | cleared. |
| Hor. Par. | | 57. | 24 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.7929 | 9.8985 | 0°. | 19.′ | 55″ |
| 10.6664 | 10.4628 | 0. | 24. | 58 |
| 0.4964 | 0.4964 | 0. | 5. | 3 |
| 0.9557 | 0.8577 | 38. | 27. | 20 |
| | | + | 1. | 32 |
| | True Diſtance | 38. | 28. | 52 |

### EXAMPLE XII.

| | | | | |
|---|---|---|---|---|
| Diſtance | 45°. | 40.′ | 14″ | corrected. |
| Star's Altitude | 15. | 54. | 4 | cleared. |
| Moon's Alt. | 21. | 26. | 4 | cleared. |
| Hor. Par. | | 56. | 30 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.8545 | 10.0101 | 0°. | 21.′ | 39″ |
| 10.5623 | 10.4372 | 0. | 20. | 11 |
| 0.5032 | 0.5032 | 0. | 1. | 28 |
| 0.9200 | 0.9505 | 45. | 38. | 46 |
| | | + | 1. | 20 |
| | True Diſtance | 45. | 40. | 6 |

### EXAMPLE XIII.

| | | | | |
|---|---|---|---|---|
| Diſtance | 65° | 27.′ | 30″ | corrected. |
| Star's Altitude | 78. | 18. | 6 | cleared. |
| Moon's Alt. | 15. | 21. | 6 | cleared. |
| Hor. Par. | 1. | 0. | 25 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.9589 | 10.3405 | 1° | 5.′ | 2″ |
| 10.0091 | 10.5786 | 0. | 7. | 16 |
| 0.4741 | 0.4741 | 0. | 57. | 46 |
| 0.4421 | 1.3932 | 64. | 29. | 44 |
| | | + | 3. | 17 |
| | True Diſtance | 64. | 33. | 1 |

### EXAMPLE XIV.

| | | | | |
|---|---|---|---|---|
| Diſtance | 49° | 17.′ | 21″ | corrected. |
| Star's Altitude | 27. | 15. | 5 | cleared. |
| Moon's Alt. | 43. | 20. | 5 | cleared. |
| Hor. Par. | | 58. | 50 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.8796 | 10.0652 | 0° | 35.′ | 33″ |
| 10.3393 | 10.1635 | 0. | 34. | 45 |
| 0.4856 | 0.4856 | 0. | 0. | 48 |
| 0.7045 | 0.7143 | 49. | 16. | 33 |
| | | + | 1. | 20 |
| | True Diſtance | 49. | 17. | 53 |

### EXAMPLE XV.

| | | | | |
|---|---|---|---|---|
| Diſtance | 35° | 29.′ | 45″ | corrected. |
| Star's Altitude | 17. | 40. | 5 | cleared. |
| Moon's Alt. | 20. | 34. | 5 | cleared. |
| Hor. Par. | | 56. | 24 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.7640 | 9.8533 | 0° | 29.′ | 28″ |
| 10.5179 | 10.4543 | 0. | 27. | 46 |
| 0.5040 | 0.5040 | 0. | 1. | 42 |
| 0.7859 | 0.8116 | 35. | 28. | 3 |
| | | + | 1. | 11 |
| | True Diſtance | 35. | 29. | 14 |

### EXAMPLE XVI.

| | | | | |
|---|---|---|---|---|
| Diſtance | 102° | 30.′ | 0″ | corrected. |
| Star's Altitude | 15. | 25. | 5 | cleared. |
| Moon's Alt. | 27. | 30. | 5 | cleared. |
| Hor. Par. | | 57. | 3 | ſame time. |

### OPERATION.

| | | | | |
|---|---|---|---|---|
| 9.9896 | 10.6542 | 0° | 15.′ | 32″ |
| 10.5754 | 10.3356 | 0. | 5. | 51 |
| 0.4990 | 0.4990 | 0. | 21. | 23 |
| 1.0640 | 1.4888 | 102. | 8. | 37 |
| | | + | 2. | 32 |
| | True Diſtance | 102. | 11. | 9 |

## Second Method.

1. Clear the Alt.$^{s}$ from Sem.$^{r}$ Dip & Ref.$^{n}$

2. Find C for Par.$^{x}$ 3. R. for Ref.$^{n}$ & F.

4. True Distance. 5. Time at Greenw.

6. Put Co-lat. Pol. dist. & their Co-ar.$^{s}$
Co-alt. Sum & ½ Sum, as 1.$^{st}$ Method,

7. Sub. Co-lat. & Pol. dist. from ½ Sum,
the Rem.$^{rs}$ give their two Sines.

8. Add together the two Co-ar.$^{s}$ & Sines;
Half the Sum is Sine of an Arch,
Twice this Arch is the Time to Noon.

Published :By S. Dunn, Dec.$^{r}$ 1. 1790. Fleet Street.

### EXAMPLE XVII.

|  |  |  |  |  |
|---|---|---|---|---|
| Diſtance | 33.° | 15.′ | 0″ | norrected. |
| Star's Altitude | 48. | 20. | 5 | cleared. |
| Moon's Alt. | 64. | 30. | 5 | cleared. |
| Hor. Par. |  | 55. | 29 | ſame time. |

### OPERATION.

|  |  |  |  |  |
|---|---|---|---|---|
| 9.7390 | 9.8167 | 1.° | 15.′ | 36″ |
| 10.1267 | 10.0445 | 1. | 16. | 23 |
| 0.5111 | 0.5111 | 0. | 0. | 47 |
| 0.3768 | 0.3723 | 33. | 15. | 47 |
|  |  | + | 0. | 41 |
|  | True Diſtance | 33. | 16. | 28 |

### EXAMPLE XVIII.

|  |  |  |  |  |
|---|---|---|---|---|
| Diſtance | 56.° | 17.′ | 44 | corrected. |
| Star's Altitude | 53. | 13. | 8 | cleared. |
| Moon's Alt. | 64. | 38. | 4 | cleared. |
| Hor. Par. | 1. | 1. | 9 | ſame time. |

### OPERATION.

|  |  |  |  |  |
|---|---|---|---|---|
| 9.9201 | 10.1759 | 0.° | 58.′ | 52″ |
| 10.0964 | 10.0440 | 0. | 36. | 51 |
| 0.4689 | 0.4689 | 0. | 22. | 1 |
| c.4854 | 0.6888 | 55. | 55. | 43 |
|  |  | + | 1. | 1 |
|  | True Diſtance | 55. | 56. | 44 |

### EXAMPLE XIX.

|  |  |  |  |  |
|---|---|---|---|---|
| Diſtance | 113.° | 19.′ | 26″ | corrected. |
| Sun's Altitude | 4. | 56. | 5 | cleared. |
| Moon's Alt. | 29. | 38. | 5 | cleared. |
| Hor. Par. |  | 56. | 29 | ſame time. |

### OPERATION.

|  |  |  |  |  |
|---|---|---|---|---|
| 9.9630 | 10.3653 | 0.° | 5.′ | 18″ |
| 11.0655 | 10.3059 | 0. | 12. | 3 |
| 0.5033 | 0.5033 | 0. | 17. | 21 |
| 1.5318 | 1.1745 | 113. | 2. | 5 |
|  |  | + | 6. | 7 |
|  | True Diſtance | 113. | 8. | 12 |

### EXAMPLE XX.

|  |  |  |  |  |
|---|---|---|---|---|
| Diſtance | 101.° | 46.′ | 43″ | corrected. |
| Sun's Altitude | 56. | 16. | 0 | cleared. |
| Moon's Alt. | 17. | 47. | 0 | cleared. |
| Hor. Par. |  | 56. | 40 | ſame time. |

### OPERATION

|  |  |  |  |  |
|---|---|---|---|---|
| 9.9907 | 10.6807 | 0.° | 48.′ | 9″ |
| 10.0801 | 10.5151 | 0. | 3. | 36 |
| 0.5020 | 0.5020 | 0. | 51. | 45 |
| 0.5728 | 1.6978 | 100. | 54. | 58 |
|  |  | + | 3. | 18 |
|  | True Diſtance | 100. | 58. | 16 |

From theſe Examples and their Solutions, it appears that there are two diſtinct Methods of Solution for the Effects of Refraction and Parallax, by the original Linear Tables that were invented by me; whilſt there is but one by the other Methods that have been publiſhed. That their Operations depend on different Principles, and produce very nearly the ſame Effects. That they may be eaſily applied in either of theſe two Methods, and may be uſed as mutual Proofs to each other, by two Perſons computing the ſame Data, or by one Perſon only, where any doubt ariſes concerning the Truth, and that there will then be but a few Minutes extra Application to have Certainty.

*The Reſults by the two Methods, and by a Mean of the beſt other Methods, are thus.*

| *Examp.* | *Firſt Method.* | *Second Method.* | *Means.* |
|---|---|---|---|
| 1. | 51°. 9′. 54″ | 51°. 9′. 54″ | 51°. 9′. 52″ |
| 2. | 89. 29. 16 | 89. 29. 16 | 89. 29. 16 |
| 3. | 109. 34. 30 | 109. 34. 30 | 109. 34. 30 |
| 4. | 50. 26. 30 | 50. 26. 30 | 50. 26. 30 |
| 5. | 28. 8. 57 | 28. 8. 56 | 28. 8. 57 |
| 6. | 58. 43. 29 | 58. 43. 28 | 58. 43. 28 |
| 7. | 43. 31. 1 | 43. 30. 57 | 43. 31. 3 |
| 8. | 28. 58. 8 | 28. 58. 8 | 28. 58. 9 |
| 9. | 62. 27. 27 | 62. 27. 24 | 62. 27. 26 |
| 10. | 85. 2. 26 | 85. 2. 29 | 85. 2. 25 |
| 11. | 38. 28. 55 | 38. 28. 52 | 38. 28. 55 |
| 12. | 45. 40. 6 | 45. 40. 6 | 45. 40. 5 |
| 13. | 64. 32. 57 | 64. 33. 1 | 64. 32. 59 |
| 14. | 49. 17. 55 | 49. 17. 53 | 49. 17. 54 |
| 15. | 35. 29. 15 | 35. 29. 14 | 35. 29. 16 |
| 16. | 102. 11. 12 | 102. 11. 9 | 102. 11. 10 |
| 17. | 33. 16. 29 | 33. 16. 28 | 33. 16. 30 |
| 18. | 55. 56. 44 | 55. 56. 44 | 55. 56. 45 |
| 19. | 113. 8. 14 | 113. 8. 12 | 113. 8. 15 |
| 20. | 100. 58. 15 | 100. 58. 16 | 100. 58. 15 |

Moreover, it is farther obſerveable, that when either the Sun or Star have great Altitude, and likewiſe the Moon, both of which are always preferable, on account of the variable ſtate of Refraction in ſmall Altitudes, a great part of the joint effects of Refraction and final correction for Parallax in Diſtance, may be taken from Linear Table III. by Inſpection, and this will reduce the Computus very much; for when the two Altitudes will not go into Linear Table I. neither it nor Linear Table II. are to be uſed, but the effect of Refraction from Linear Table III. and when the Diſtance and Moon's Altitude are great, the final correction for Parallax is but little. What the Error's would be in uſing ſuch an Approximation in the former Examples, appears by their Solutions and that Table.

## IX. *Of the Firſt and Second Methods for Refraction and Parallax, and their Agreement with a Medium of the other Methods.*

The agreement there is between the Firſt and Second Methods and a medium of other true Methods, plainly ſhews that they are founded on true Principles, and that the difference between their concluſions and other Methods, the beſt that have been publiſhed, is too ſmall to leſſen their Utility. It is to be obſerved, that in all theſe Anſwers, the Numbers have been taken out of the Tables roughly as they have occurred, without regard to parts of Minutes of a Degree, and as they are to be calculated in real Practice; otherwiſe theſe two Methods might have approximated the truth itſelf, without any perceptible Error.

It is here ſuppoſed, that the whole correction for Refraction, has been made by uſing Tables I. and II. when they have been wanted, and Table III. when I. could not be applied, in order to have this accuracy; but if that be thought too much trouble, if the Altitudes are great and Diſtance ſo too; in the wide Oceans, where it is certain that no Dangers are near, an approach may be had toward the true Diſtance by taking the Seconds in Table III. for the effect of Refraction, ſuppoſing the final correction of Parallax to vaniſh. By this, under the forementioned circumſtances, and no other, the Error may be half, a third, or quarter of a Degree of Longitude at Sea. This Mode of allowance ſhould not be uſed, but by ſuch as have thorough Inſight in the nature of the Problem, or ſuch as cannot do the whole, as Truth itſelf is always preferable.

### X. *Of the Data whereby the Computation for Refraction by the Second Method may be ſhortened and taken out by Inſpection.*

The Data whereby this computation may be ſhortened, are ſuch Altitudes as will not go into Table I, for then II. is not to be uſed, but III. gives the correction for Refraction by Inſpection, and in order to know when ſuch opportunities will happen, it is neceſſary to conſider what different Poſitions ariſe by the apparent diurnal Motion among the circles of the Sphere, thus.

At any place at Sea, frequent opportunities happen, when the Luminaries may be obſerved not only at equal Altitudes, but when they have certain differences of Altitude amounting to but a few degrees. At ſuch times, 1. There will be the leaſt Error in the Longitude found, through a different Refraction. 2. The correction for Refraction appears at ſight, and final correction for Parallax frequently vaniſhes. 3. The Objects are beſt to be obſerved. 4. The Obſervations may be made, firſt, when a weſtermoſt object is higheſt, ſecondly, when it is nearly of equal Altitude with the other, and thirdly, when it is loweſt of nearly the ſame Altitude as the moſt eaſtwardly was at firſt. By the Mean of theſe three Reſults, the Error of a different Refraction becomes annihilated, for the Errors counteract each other; and thus may the Longitudes of Places on Land, and near the Coaſts, be correctly aſcertained by the Lunar Method. By ſuch Reſults, the Geography of the Lands, and Hydrography of the Coaſts may be corrected, but not by Obſervations and Calculations that are imperfect. The leſſer and greater Altitudes of this kind, are as follows.

XI.

## XI.

In the following Table, find the leſſer of the two obſerved Altitudes, under Leſs. Alt. and if the greater of the two obſerved Altitudes doth not exceed the number of Degrees oppoſite to it under Gr. Alt. the effect of Refraction may be taken from Table III. by Inſpection.

| Leſs. Alt. | Gr. Alt. | Leſs. Alt. | Gr. Alt. | Leſs. Alt. | Gr. Alt. | Leſs. Alt. | Gr. Alt. |
|---|---|---|---|---|---|---|---|
| 16° | 21° | 31° | 41° | 46° | 69° | 61° | 90° |
| 17. | 22. | 32. | 43. | 47. | 72. | 62. | 90. |
| 18. | 24. | 33. | 45. | 48. | 75. | 63. | 90. |
| 19. | 25. | 34. | 46. | 49. | 80. | 64. | 90. |
| 20. | 27. | 35. | 48. | 50. | 90. | 65. | 90. |
| 21. | 28. | 36. | 50. | 51. | 90. | 66. | 90. |
| 22. | 29. | 37. | 51. | 52. | 90. | 67. | 90. |
| 23. | 31. | 38. | 53. | 53. | 90. | 68. | 90. |
| 24. | 32. | 39. | 55. | 54. | 90. | 69. | 90. |
| 25. | 33. | 40. | 56. | 55. | 90. | 70. | 90. |
| 26. | 35. | 41. | 58. | 56. | 90. | 71. | 90. |
| 27. | 36. | 42. | 60. | 57. | 90. | 72. | 90. |
| 28. | 38. | 43. | 62. | 58. | 90. | 73. | 90. |
| 29. | 39. | 44. | 64. | 59. | 90. | 74. | 90. |
| 30. | 40. | 45. | 66. | 60. | 90. | 75. | 90. |

The great variety of Altitudes and Diſtances, that come within theſe Limits, and the eaſe with which the Obſervations may frequently be made within them, ought to engage the Obſerver's attention. Then, for great Altitudes of the Moon, and great Diſtances, the laſt correction for Parallax will be nothing.

## XII. *Of Time at the Ephemeris's Place.*

This is a very eaſy Proceſs and beſt performed alike in both Methods. It may be proper to conſider what ſubdiviſions are neceſſary for the performance of this part, by the help of Delineations.

In making this calculation, an Error of two Seconds, any how in the diſtance of Sun and Moon, or Moon and Star, evidently produces an Error of one Mile in the Longitude, at the Equinoctial; and an Error of twenty Seconds in Diſtance produces Error in Longitude ten Miles. The Proportions are for all Diſtances, to near a Degree and Half; and therefore, a Scale of ſimilar Triangles for working this proportionality, would neither be accurate enough nor convenient for this purpoſe. Beſides, when a Scale is applied for Proportionality, the Operator has ſeveral things to think of, that are not eaſy to expert Performers, the place of a point moving this way, that way, and a third way, for all this is abſolutely neceſſary, much more difficult than any thing by the logiſtics, and after all much leſs correct.

## XIII. *Of Time at the Ship's Place.*

As a knowledge of Time at the Ephemeris's Place is the firſt great ſtep toward the diſcovery of the Longitude by the Sun and Moon, or Moon and Zodical Stars, ſo is the knowledge of Time at the Ship's Place the ſecond great ſtep, and the Sum or Difference of theſe two times is the Longitude.

There are ſo many ways whereby the Time at the Ship or Ship's Time may be knowu, that it is too large a ſubject to be particularized here, and not neceſſary as they have all been treated of with Improvements, in my other works on theſe Subjects.

Were there certainty in ſailing by Courſe and Diſtance, and in carrying Time without Error, Rules might be made for anſwering all Caſes, but then Difficulties would occur, and therefore it will be beſt to take that Aſſiſtance which is always eaſily had from the Journal of a Ship at Sea. In the Journal, the Ship's Latitude is carried on from Noon to Noon, and the Reckoning will eaſily ſhew the Difference of Latitude, at intermediate times; this gives the Latitude at the time of Obſervation for the Longitude by the Ship's Clock, as has been already mentioned, and is one of the Data or things ſuppoſedly known. It is needleſs to mention, that Obſervers or Computors ſhould have the beſt Information that can be had concerning this Article.

In order to have the Longitude determined as correct as the Obſervations and Tables will admit, every one of the Data ſhould be as correct as poſſible. A ſingle Minute of a Degree too much or too little, in the Latitude, Declination or Altitude, of either Luminary, will ſometimes cauſe an Error of ſeveral Miles in the Longitude found, and thereby that utility in Navigation, Hydrography and Geography which is intended, will be leſſened. The Logarithmic Computus is the only means whereby this, or even a much leſs accuracy can be expected.

As the Latitude and Altitude ſhould be known to the neareſt Minute of a Degree, ſo ſhould the Declination be taken from the Ephemeries for the Time of Obſervation to the like accuracy. Then, if the Latitude and Declination are both North or both South, ſubtract the Declination from ninety Degrees, the remainder is the Polar-diſtance; but, if the Latitude and Declination are one North and the other South, add the Declination to ninety Degrees, the ſum is the Polar-diſtance. Having the Co-latitude, the Polar-diſtance, and Sun's Co-altitude, the Time from Noon will be found by the firſt and ſecond Method, to the neareſt Half Mile, as follows.

In theſe Examples, the three firſt Terms in Degrees and Minutes are the Co-latitude, Polar-diſtance, and Co-altitude, the other Figures, are thoſe of the whole Operation.

### EXAMPLE I.

First Method.

*For Time at the Ship.*

| | | | | |
|---|---|---|---|---|
| | 38°. | 32′ | 0.20553 | |
| | 23. | 28 | 0.39988 | |
| | 42. | 24 | 9.89771 | |
| ½ Sum | 52. | 12 | 9.23098 | |
| Rem. | 9. | 48 | 19.73410 | Sum. |
| | 42. | 35 | 9.86705 | |
| | 85. | 10 | Time from Noon. | |

Second Method.

*For Time at the Ship.*

| | | | | |
|---|---|---|---|---|
| | 38°. | 32′ | 0.20553 | |
| | 23. | 28 | 0.39988 | |
| | 42. | 24 | 9.37341 | |
| ½ Sum | 52. | 12 | 9.68191 | |
| 1ſt Rem. | 13. | 40 | 19.66073 | Sum. |
| 2d Rem. | 28. | 44 | | |
| | 42. | 35 | 9.83036 | |
| | 85. | 10 | Time from Noon. | |

EXAMPLE II.

## EXAMPLE II.

First Method.

*For Time at the Ship.*

| | | |
|---|---|---|
| | 54° 48′ | 0.08770 |
| | 112. 11 | 0.03340 |
| | 64. 51 | 9.95397 |
| ½ Sum | 115. 55 | 9.89091 |
| Rem. | 51. 4 | 19.96598 Sum. |
| | 15. 56 | 9.98299 |
| | 31. 52 | Time from Noon. |

Second Method.

*For Time at the Ship.*

| | | |
|---|---|---|
| | 54° 48′ | 0.08770 |
| | 112. 11 | 0.03340 |
| | 64. 51 | 9.94231 |
| ½ Sum | 115. 55 | 8.81367 |
| 1ſt Rem. | 61. 7 | 18.87708 |
| 2d Rem. | 3. 44 | |
| | 15. 56 | 9.43854 |
| | 31. 52 | Time from Noon. |

When either of theſe Methods are uſed in finding Time at the Ship, four figures of Logarithms give the anſwer more correct than inſtrumental Methods of Solution, and ſix figures (which are a hundred times as exact) work Altitudes taken very near the Meridian itſelf. Theſe are advantages peculiar to the logarithmic Proceſs, previous to which any preparation of the Data may be made, that the nature of the ſubject requires, and Accuracy be preſerved with the greateſt eaſe, for all Altitudes, all Declinations, and all Latitudes, from the Equinoctial to the Poles. This the Lunar Method requires, and it would be deſtructive of this Science, to uſe ineffectual Means for theſe important Purpoſes. There are other ſmall Corrections which may be ſeen at large in my other works on this Subject.

In finding Time at the Ship by the Stars, the Altitude is to be taken and cleared from Dip and Refraction; then, by the Latitude, Polar-diſtance and Co-altitude, the true time of the Star's being either paſt the Meridian or ſhort of it, is to be found, as if it was the Sun. The Difference between the Sun and Star's Right Aſcenſions at the Time of Obſervation is their Equatorial-diſtance. Then, the place of the Star, of the Sun and of the Meridian, will plainly ſhew their ſituations. The Difference of Right Aſcenſions applied to the Star's Equatorial Diſtance from the Meridian, by adding or ſubtracting, gives the Solar-time at the Ship paſt Noon or ſhort of Noon.

## XIV. *Of Longitude at the Ship's Place.*

Having found the Time at the Ephemeris's Place, and the Time at the Ship's Place, the Ship's Longitude is had by adding or ſubtracting, thus. 1. If the Obſervation be made After-noon at the Ship, ſubtract one Time from the other, the Remainder is the Longitude of the Ship from the Ephemeris's Place; and it is Weſt Longitude when the Time at the Ephemeris's place is greateſt, but otherwiſe Eaſt Longitude. 2. If the Obſervation be made Before-noon at the Ship, add the two Times together, their Sum is Weſt Longitude from the Ephemeris's Place; and it's Remainder to 360 Degrees is Eaſt Longitude.

## XV. *Of Auxiliary Data.*

In the Lunar Method, when either the Latitude, or one of the two Altitudes of the cotemporary Obſervations cannot be known, it is poſſible to ſupply their places, by help of ſome other things that are known, and which may be called auxiliary Data. Thus, 1. When the Latitude is not known, a Watch that has been ſet to Time is commonly uſed to ſhew the Time at the Ship. 2. When either of the two Altitudes has been incorrect, Rules have been uſed for inferring it. Theſe things are contained in my other Works, and others might be added, but not without introducing greater Difficulties than will ever attend the Method by actual cotemporary Obſervations.

Cotemporary Obſervations in the Lunar Method, have ſure and eaſy ways for computing the true Diſtance of Sun and Moon, or Moon and Star, without an Error that would effect the Longitude one Mile. They have the like certainty when the Altitudes have been correctly taken. The like for Time at the Ship, when the Co-latitude, Polar-diſtance and Co-altitude have been ſettled as correct as the Tables and Inſtruments will eaſily make them. This is an accuracy which the Lunar Method of Calculation admits of, and which is neceſſary to be kept to by Computors, although leſs accuracy be ſufficient for correcting the Errors that ariſe in Practical Navigation; becauſe, the Errors through imperfection in Inſtruments and Obſervers, through unfavourable Weather, the Ship's Motion and other Cauſes, are enough beſides, to counteract the moſt perfect Obſervations, Tables and Calculations.

## XVI. CONCLUSION.

Thus, the Longitude Science has three diſtinct parts, which have been explained in this work. 1. The true Diſtance of Centres which determines the Time for the Ephemeris's Meridian, from the Equinoctial to the Poles. 2. The two Altitudes of Sun and Moon, or of Moon and Star, which (beſides their uſe in finding the effects of Refraction and Parallax) would, with their Right Aſcenſions and Declinations for the time of Obſervation, determine the Latitude of the place, when the cotemporary obſervations are made. 3. That Latitude, which with the Sun or Star's Altitude, and its Declination, determines the Time at the Meridian of the Ship. Theſe are the two times that give the Ship's Longitude.

Hence, the Longitude might be taken without Altitudes as in the three cotemporary Obſervations; for the obſerved Diſtance, the Latitude of the Ship, and included Hour-angles of Sun and Star, do limit the Altitudes and would determine them; but herein a great ſtreſs lays on the Moon's Parallax and difficult Computations, which the Altitudes do not require, and the Time-keeper muſt keep Time as perfect as the Heavens, otherwiſe the Hour-angles will be wrong, and there will be Danger. Now, the greateſt Difficulties ariſe for want of a clear Horizon, and that Defect may be in a great meaſure ſupplied, by Methods that have been tried and found practicable at Sea. In favourable Weather the Altitudes may be had to the greateſt Accuracy, whilſt by other ways the Data would be imperfect.

N. B. When the laſt Proper Log. is found in its Table, a fourth of the Side-ſeconds are Minutes of a Degree, theſe together with the bottom Degrees and Minutes and the firſt Hours in Degrees, makes the Ephemeris's Time.

*Page* 8 *read*, inevitably. at *a Medium*. 10. *read*, *lower* or *upper Limb*.

E N D.